

Mass bound of the lightest neutral Higgs scalar in the extra U(1) models

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The upper mass bound of the lightest neutral Higgs scalar is studied in the μ problem solvable extra U(1) models by using the analysis of the renormalization group equations. In order to restrict the parameter space we take account of a condition of the radiative symmetry breaking and some phenomenological constraints. We compare the bound obtained based on this restricted parameter space with the one of the next to the minimal supersymmetric standard model (NMSSM). Features of the scalar potential and renormalization group equations of the Yukawa couplings among Higgs chiral supermultiplets are rather different between them. They can reflect in this bound.

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1. Introduction

Low energy supersymmetry is one of the main subjects of present particle physics. It is considered to solve a weak scale stability problem called the gauge hierarchy problem in the standard model (SM). Although we donot have any direct evidence for it, it has been stressed that the gauge coupling unification occuring in a rather precise way in the minimal supersymmetric extension (MSSM) of the SM may be an encouraging sign for the presence of the low energy supersymmetry. In the MSSM its phenomenology crucially depends on the soft supersymmetry breaking parameters and then it seems to be difficult to make useful predictions unless we know how the supersymmetry breaks down. However, there is an important exception that the lightest neutral Higgs scalar mass cannot be so heavy and it is mainly controled by the feature of the weak scale symmetry breaking [1]. This is not heavily dependent on the feature of the soft supersymmetry breaking parameters at least at the tree level. Thus the knowledge of its possible upper bound is crucial to judge the validity of the low energy supersymmetry from a viewpoint of the energy front of the accelerator experiment. This aspect has been extensively studied taking account of a radiative correction mainly due to a large top Yukawa coupling [2].

It is well known that there still remains a hierarchy problem called μ problem in the MSSM. Why a supersymmetric Higgsino mixing term parametrized by μ is a weak scale cannot be explained in the MSSM [3]. A simple and promising candidate for its solution is an extension of the MSSM by the introduction of an extra U(1) gauge symmetry and a SM singlet field S with a nonzero charge of this extra U(1) [4, 5]. The essential feature of this model is described by the following superpotential

$$W_{U(1)'} = \lambda S H_1 H_2 + k S \bar{g} g + h_t Q H_2 \bar{T} + \cdots, \quad (1)$$

where H_1 and H_2 are usual doublet Higgs chiral superfields and the ellipses stand for the remaining terms in the MSSM superpotential other than the μ term and the top Yukawa coupling. In the second term g and \bar{g} stand for the extra color triplet chiral superfields which are important to induce the μ scale. In the superpotential $W_{U(1)'}$ we also explicitly write the top Yukawa coupling because of its importance in the electroweak radiative symmetry breaking as in the case of the MSSM [6].

The vacuum of these models is parametrized by the vacuum expectation values (VEVs)

of Higgs scalar fields such as

$$\langle H_1 \rangle = \begin{pmatrix} v_1 \\ 0 \end{pmatrix}, \quad \langle H_2 \rangle = \begin{pmatrix} 0 \\ v_2 \end{pmatrix}, \quad \langle S \rangle = u, \quad (2)$$

where v_1 and v_2 are assumed to be positive and $v_1^2 + v_2^2 = v^2 (\equiv (174\text{GeV})^2)$ should be satisfied¹. The vacuum in this model is parametrized by $\tan \beta = v_2/v_1$ and u . The extra U(1) symmetry is assumed to be broken at the region not far from the weak scale by a VEV of the scalar component of S because of the radiative effect caused by the second term in $W_{U(1)}$ [4, 5] and then the μ scale is induced as $\mu = \lambda u$. Thus in this model the sign of μ is fixed as the one of u automatically.

This extra U(1) symmetry forbids a bare μ term in the superpotential and simultaneously makes the model free from the massless axion and tadpole problems. These features seem to make this model more promising than the next to the minimal supersymmetric standard model (NMSSM) [7] which is similar to this extra U(1) model but is extended only by a SM singlet chiral superfield S with the superpotential

$$W_{\text{NMSSM}} = \lambda S H_1 H_2 + \frac{1}{3} \kappa S^3 + h_t Q H_2 \bar{T} + \dots \quad (3)$$

It is also interesting that this kind of extra U(1) models can be often obtained as the effective models of a lot of superstring models [8]. Various interesting features of this type of models have been studied in many works by now [9–12]. Among the phenomenology of these models the lightest neutral Higgs scalar mass is also an important target for the detailed investigation. Of course, also in these models the lightest neutral Higgs scalar can be expected to be generally not so heavy. The interesting point is that its upper bound can be calculable with no dependence on the soft supersymmetry breaking parameters at least at the tree level as in the case of the NMSSM [13–17]. The dependence on the soft supersymmetry breaking parameters comes in through the loop correction mainly due to the large top Yukawa coupling and the second term of Eq. (1).

In this paper we estimate the upper bound m_{h^0} of this lightest neutral Higgs scalar mass on the correct vacuum. The correct vacuum is determined as the radiatively induced minimum of the effective potential in the suitable parameter space. In this approach we

¹ In the following discussion we do not consider the spontaneous CP violation. Under this assumption the sign of u cannot be fixed freely but it should be dynamically determined by finding the potential minimum.

use the one-loop effective potential and solve the relevant renormalization group equations (RGEs) numerically. We will pay our attention on the comparison of this upper bound with the one of the NMSSM within the phenomenologically allowable parameter region. In the NMSSM it has been known through many works that the triviality bound of a Yukawa coupling λ of Higgs chiral superfields strictly control the upper bound of the lightest neutral Higgs mass [15, 16]. Our approach is somehow different from this usual one. We find the phenomenologically acceptable parameter subspace in the rather wide parameter space by taking account of the radiative symmetry breaking condition and some phenomenological conditions such as the chargino mass and the charged Higgs scalar mass *etc.*. The estimation of the upper mass bound of the lightest neutral Higgs scalar is carried out in this restricted parameter subspace. Although the result of this approach is necessarily dependent on the assumption for the soft supersymmetry breaking parameters, we consider that it is possible to obtain the useful results by studying the wide region of the parameter space.

2. Extra U(1) models

In this section we discuss more detailed features of the extra U(1) models and give the basis of the present study. Since the NMSSM is well known and discussed in many papers[], it is convenient to explain the points by using the extra U(1) models. The superpotential of our considering extra U(1) models is defined by Eq. (1). Soft supersymmetry breaking parameters are introduced as

$$\begin{aligned} \mathcal{L}_{\text{soft}} = & - \sum_i m_{\phi_i}^2 |\phi_i|^2 + \left(\sum_a \frac{1}{2} M_a \bar{\lambda}_a \lambda_a + \text{h.c.} \right) \\ & + \left(A_\lambda \lambda S H_1 H_2 + A_k k S \bar{g} g + A_t h_t Q H_2 \bar{T} + \text{h.c.} \right), \end{aligned} \quad (4)$$

where the first two terms are mass terms of the scalar component ϕ_i of each chiral supermultiplet and of gauginos λ_a . We use the same notation for the scalar component as the one of the chiral superfield to represent the trilinear scalar couplings in the last parentheses. Other freedoms remaining in the models are extra matter contents and a type of extra U(1). On these points we confine our study into the typical extra U(1) models derived from E_6 , which are listed in Table 1. At the TeV region they are assumed to have only one extra U(1) symmetry which is broken only by the VEV of S and give a solution to the μ problem [11]. As discussed in Ref. [16] for the case of NMSSM, the extra matter contents

	Q	\bar{U}	\bar{D}	L	\bar{E}	H_1	H_2	g	\bar{g}	S	N
SM	(3,2)	(3*,1)	(3*,1)	(1,2)	(1,1)	(1,2)	(1,2)	(3,1)	(3*,1)	(1,1)	(1,1)
Y	$\frac{1}{3_2}$	$-\frac{4}{3}$	$\frac{2}{3}$	-1	2	-1	1	$-\frac{2}{3}$	$\frac{2}{3}$	0	0
Q_η	$-\frac{2}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{1}{3}$	$-\frac{2}{3}$	$\frac{1}{3}$	$\frac{4}{3}$	$\frac{4}{3}$	$\frac{1}{3}$	$\frac{5}{3}$	$-\frac{5}{3}$
$Q_{\xi\pm}$	$\pm\frac{1}{\sqrt{6}}$	$\pm\frac{1}{\sqrt{6}}$	$\pm\frac{2}{3}$	$\pm\frac{1}{\sqrt{6}}$	$\pm\frac{1}{\sqrt{6}}$	$\mp\frac{1}{\sqrt{6}}$	$\mp\frac{1}{\sqrt{6}}$	$\mp\frac{1}{\sqrt{6}}$	$\mp\frac{1}{\sqrt{6}}$	$\pm\frac{1}{\sqrt{6}}$	0

Table 1 The charge assignment of extra U(1)s which are derived from E_6 [12]. These charges are normalized as $\sum_{i \in \mathbf{27}} Q_i^2 = 20$.

affect indirectly the low energy value of the Yukawa coupling λ through the influence on the running of the top Yukawa coupling. This is rather important to estimate the Higgs mass bound. In the present model such kind of effects on the Yukawa couplings may also be expected but its effect is more complicated than the NMSSM as discussed later. If we introduce the extra field contents arbitrarily, the cancellation of the gauge anomaly may require to introduce the additional fields which again affect the running of Yukawa coupling λ and so on. Thus for the estimation of the Higgs mass bound it is important to fix the matter contents in the anomaly free way in the present study.

As the matter contents we assume the MSSM contents and additional extra matter fields

$$\left[3(Q, \bar{U}, \bar{D}, L, \bar{E}) + (H_1, H_2)\right]_{\text{MSSM}} + 3(g, \bar{g}) + 2(H_1, H_2) + 3(S) + 3(N),$$

which can be derived from three $\mathbf{27}$ s of E_6 shown in Table 1. This set satisfies the anomaly free conditions. We can also add extra fields to these in the form of vector representations constructed from the fields listed in Table 1. Here we consider the following two cases as the additional extra chiral superfields

$$(A) \quad (H_a) + (H_a^*), \quad (B) \quad (g + H_a + H_b) + (g^* + H_a^* + H_b^*),$$

where $a, b = 1$ or 2 and the fields in the second parentheses come from $\mathbf{27}^*$ of E_6 . At least on the sector of $SU(3)_C \times SU(2)_L \times U(1)_Y$ these matter contents are the same as the one of $[\text{MSSM} + n(\mathbf{5} + \mathbf{5}^*)]$ where $\mathbf{5}$ and $\mathbf{5}^*$ are the representations of the usual $SU(5)$. The case (A) corresponds to $n = 3$ and (B) to $n = 4$. The $n = 3$ is the critical value for the one-loop β -function of $SU(3)$. It makes this one-loop β -function be zero. The interesting point of these field contents is that the unification scale of $SU(3)_C \times SU(2)_L \times U(1)_Y$ is not

shifted from the MSSM one. The $n = 4$ case saturates the β -function for the perturbative running of gauge couplings up to the unification scale $\sim 3 \times 10^{16}$ GeV. Although this addition seems to be artificial, this type of spectrum can be expected in the Wilson line breaking scenario of the E_6 type superstring model. We use these contents to compare the feature between the NMSSM and our extra U(1) models².

The existence of multi-generation extra fields brings an ambiguity in Eq. (1). The coupling λ and k can have generation indices for extra fields such as S , H_1 , H_2 , g and \bar{g} . On this point we make the following assumption to make the argument simple.³ Only one S can have the couplings in Eq. (1) and one pair of (H_1, H_2) corresponding to the one of the MSSM alone gets the VEVs. Extra colored singlets (g_i, \bar{g}_i) have a diagonal coupling to this S as $k_i S g_i \bar{g}_i$, where all the coupling constant k_i show the same behavior in the RGEs because (g_i, \bar{g}_i) are completely symmetric for the generation index i in the models. The fermion components of g_i and \bar{g}_i can get mass through this coupling.⁴ On the other hand, the fermion components of the remaining S which do not couple to the usual Higgsinos in H_1 and H_2 can get their masses through the one-loop correction. From a viewpoint of the model construction, the serious phenomenological problem will be how the fermion components in other remaining extra matter fields can get their masses. Although they can be generally massive through the gaugino mediated one-loop diagrams, their magnitude seems not to be enough to satisfy the phenomenological constraints. In the ξ_- model given in Table 1 we can introduce the intermediate scale through the D-flat direction of N and N^* whose existence does not affect our discussion in the later part of this paper. If this is the case, they can have the weak scale masses through the nonrenormalizable interactions in the superpotential such as $\frac{1}{M_{\text{pl}}} N N^* g g^*$. Although this is phenomenologically important, it can be improved by the suitable extension without changing the following results and thus we do not get involved in this point further here.

In our considering models the tree level scalar potential including the soft supersym-

²We should note that if every extra U(1) is broken near the unification scale, these models are equal to the NMSSM with the equivalent extra matters as discussed in Ref. [16].

³As far as we use Eq. (7) for the upper bound of the lightest neutral Higgs mass, this assumption seems to be reasonable. This assumption affects the RGEs of some parameters and also one-loop correction to the Higgs scalar mass. Other cases will be discussed later.

⁴If we change the charge assignment for some fields, some extra fields can be heavy at the intermediate scale as discussed in [17]. Although this kind of possibility can be realized in the ξ_{\pm} model, we do not consider it in this paper.

metry breaking terms can be written as

$$\begin{aligned}
V_0 = & \frac{1}{8} (g_2^2 + g_1^2) (|H_1|^2 - |H_2|^2)^2 + (|\lambda S H_1|^2 + |\lambda S H_2|^2) \\
& + m_1^2 |H_1|^2 + m_2^2 |H_2|^2 - (A_\lambda \lambda S H_1 H_2 + \text{h.c.}) \\
& + \frac{1}{8} g_E^2 (Q_1 |H_1|^2 + Q_2 |H_2|^2 + Q_S |S|^2)^2 + \lambda^2 |H_1 H_2|^2 + m_S^2 |S|^2,
\end{aligned} \tag{5}$$

where Q_1 , Q_2 and Q_S are the extra U(1) charges of H_1 , H_2 and S , respectively. The first two lines are found to have the corresponding terms in the MSSM if we remind the fact that μ is realized as $\mu = \lambda u$. The third line contains new ingredients. Its first term is a D-term contribution of the extra U(1) and g_E stands for its gauge coupling constant.

Potential minimum condition for Eq. (5) can be written as,

$$\begin{aligned}
m_1^2 = & -\frac{1}{4} (g_2^2 + g_1^2) (v_1^2 - v_2^2) - \frac{1}{4} g_E^2 Q_1 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S u^2) - \lambda^2 (u^2 + v_2^2) + \lambda A_\lambda u \frac{v_2}{v_1}, \\
m_2^2 = & \frac{1}{4} (g_2^2 + g_1^2) (v_1^2 - v_2^2) - \frac{1}{4} g_E^2 Q_2 (Q_1 v_1^2 + Q_2 v_2^2 + Q_S u^2) - \lambda^2 (u^2 + v_1^2) + \lambda A_\lambda u \frac{v_1}{v_2}, \\
m_S^2 = & -\frac{1}{4} g_E^2 Q_S (Q_1 v_1^2 + Q_2 v_2^2 + Q_S u^2) - \lambda^2 (v_1^2 + v_2^2) + \lambda A_\lambda \frac{v_1 v_2}{u}.
\end{aligned} \tag{6}$$

This constrains the soft SUSY breaking masses of Higgs scalars around the weak scale. As the second derivative of V_0 in Eq. (5) we can derive the mass matrix of the CP-even neutral Higgs scalar sector which is composed of three neutral components H_1^0 , H_2^0 and S . The goodness of this treatment has been discussed in the MSSM case [2] and we follow this argument. If we note the fact that the smallest eigenvalue of any matrix is always smaller than any diagonal elements, we can obtain the tree level upper bound of this lightest Higgs scalar in an independent way of the soft supersymmetry breaking parameters by transforming the basis into the suitable one. This upper bound can be written as [13, 4]

$$m_{h^0}^{(0)2} \leq m_Z^2 \left[\cos^2 2\beta + \frac{2\lambda^2}{g_1^2 + g_2^2} \sin^2 2\beta + \frac{g_E^2}{g_1^2 + g_2^2} (Q_1 \cos^2 \beta + Q_2 \sin^2 \beta)^2 \right], \tag{7}$$

where we used the potential minimization condition (6). The first two terms correspond to the ones of the NMSSM in which their behavior has been studied in many works [14, 15, 16]. The running of a coupling constant λ and its triviality bound have been shown to be crucially dependent on the extra matters [16]. The extra U(1) effect appears through the last term which is its D-term contribution. Equation (7) can show the different $\tan \beta$ dependence from the one in the MSSM depending on the value of λ and also the

type of extra U(1). In the case of MSSM the upper bound of the lightest neutral Higgs mass always increases with $\tan\beta$ in the region of $\tan\beta > 1$. If $\lambda \lesssim 0.6$, the present model also shows the same behavior. On the other hand, for the same region of $\tan\beta$ its upper bound can decrease with increasing $\tan\beta$ when $\lambda \gtrsim 0.6$ which does not depend on the model so heavily. The NMSSM shows the similar feature, which can be seen in Ref. [16]. Although this may be potentially altered by the radiative correction, it is one of the typical features coming from the $\lambda SH_1 H_2$ in these models different from the MSSM.

We should note that the bound formula Eq. (7) is applicable only in the case of $u \gg v_1, v_2$ ⁵. In the extra U(1) model the value of u can be constrained from below by the conditions on the mass of this extra U(1) gauge boson and its mixing with ordinary Z^0 . As far as we donot consider the special situation such as $\tan^2\beta \sim Q_1/Q_2$ under which the mixing with the ordinary Z^0 is negligible, the hierarchical condition $u > v_1, v_2$ should be imposed to satisfy the phenomenological constraints on the extra Z' mass and its mixing with ordinary Z^0 [12]. In the sufficiently large u case λ may be constrained into a limited range required by the successful radiative symmetry breaking at the weak scale so that $\lambda u (\equiv \mu)$ takes a suitable value. In the NMSSM this kind of constraint on λ is expected to be weaker than the one of the extra U(1) model since u has no phenomenological constraint at this stage. Anyway, we need the RGE study to check whether λ can be constrained in a substantial way by this condition.

3. The comparison of extra U(1) models and the NMSSM

It is useful to discuss some qualitative features of the extra U(1) models and the NMSSM in more detail before comparing the mass bound of the lightest neutral Higgs in both models. Although the extra U(1) models and the NMSSM have the similar feature related to the μ term, they are expected to show rather different behavior in the running of Yukawa couplings k , κ and λ . The top Yukawa coupling has the same one-loop RGE in both models as,

$$\frac{dh_t}{d\ln\mu} = \frac{h_t}{16\pi^2} \left(6h_t^2 + \lambda^2 - \frac{16}{3}g_3^2 \right). \quad (8)$$

In the present field contents g_3 takes larger value at M_X than the one of the MSSM. Even if the initial value of h_t takes the large value like $O(1)$, the β -function in Eq. (8) can be

⁵ In the case of $u < v_1, v_2$ the diagonal element corresponding to S can be smaller than the right-hand side of Eq. (7). In such a case we cannot use Eq. (7) as the bound of the lightest neutral Higgs mass. We will exclude it from our study.

small due to the cancellation between a h_t term and a g_3 term. As a result, h_t tends to stay near its initial value at the intermediate scale independently whether it starts from a large value or a small value. This feature is shared by both models. On the other hand, the one-loop RGEs of κ , k and λ are largely different from each other. They are written as, in the NMSSM,

$$\begin{aligned}\frac{d\kappa}{d\ln\mu} &= \frac{\kappa}{16\pi^2} (6\kappa^2 + 6\lambda^2), \\ \frac{d\lambda}{d\ln\mu} &= \frac{\lambda}{16\pi^2} (3h_t^2 + 2\kappa^2 + 4\lambda^2).\end{aligned}\tag{9}$$

and in the extra U(1) model,

$$\begin{aligned}\frac{dk}{d\ln\mu} &= \frac{k}{16\pi^2} \left((3N_g + 2)k^2 + 2\lambda^2 - \frac{16}{3}g_3^2 \right), \\ \frac{d\lambda}{d\ln\mu} &= \frac{\lambda}{16\pi^2} (3h_t^2 + 3N_g k^2 + 4\lambda^2),\end{aligned}\tag{10}$$

where N_g is a number of the pair of the singlet colored fields g and \bar{g} which have a coupling to S . In these RGEs we neglect the effect of gauge couplings g_2 , g_1 and g_E ⁶. At first we consider the running behavior of κ and k . Since k has an effect of g_3 , it can be rather larger at the intermediate scale than κ which has no such effect and rapidly decreases according to lowering energy. This is important to determine the value of u realized in both models, which are mainly determined by m_S^2 at the low energy region. They are controlled by the one-loop RGE as

$$\frac{dm_S^2}{d\ln\mu} = \frac{1}{8\pi^2} \left(2\kappa^2(3m_S^2 + A_k^2) + 2\lambda^2(m_S^2 + m_{H_1}^2 + m_{H_2}^2 + A_\lambda^2) \right)\tag{11}$$

in the NMSSM and

$$\frac{dm_S^2}{d\ln\mu} = \frac{1}{8\pi^2} \left(3N_g k^2(m_S^2 + m_g^2 + m_{\bar{g}}^2 + A_k^2) + 2\lambda^2(m_S^2 + m_{H_1}^2 + m_{H_2}^2 + A_\lambda^2) \right),\tag{12}$$

in the extra U(1) models. The larger k compared with κ makes m_S^2 much more negative in the extra U(1) models. The larger value of u is expected in the extra U(1) models if we remind Eq. (6).

As easily seen from the RGE of λ in the extra U(1) model, the running of λ is made fast by the existence of the second term of Eq. (1) which is needed for the successful

⁶ In these equations we cannot find the fixed ratio point other than $k = 0$ or $\lambda = 0$ as far as $N_g \neq 0$ even if we ignore g_3 . This is very different situation from the NMSSM which has been discussed in Ref. [18].

radiative symmetry breaking of these models [4]. The one-loop β -function of the coupling k has a contribution of g_3 differently from the case of κ in the NMSSM. If we start k and κ from the large values at the unification scale, this feature can keep k rather large at the intermediate region and then the running of λ can be made fast by its effect compared with the one of κ in the NMSSM. This feature tends to make the value of λ at the low energy scale smaller compared with the NMSSM case if the same initial value is adopted at least. However, the initial value of k and κ should be controlled from the requirement of the radiative symmetry breaking from our view point since they play an important role in this phenomenon. We need the numerical analysis to study this aspect in more quantitative way. The extra matter effects on the RGEs are also rather different between the NMSSM and the extra U(1) models. As far as all the couplings are within the perturbative regime, the larger number of extra matter fields make the gauge couplings at the unification scale larger. As pointed out in [16], in the NMSSM this indirectly makes the low energy value of λ larger through the smallness of h_t at the intermediate scale whose β -function in Eq. (8) is kept small there. On the other hand, in the extra U(1) models the runnings of k and λ are simultaneously affected by the extra matters in both direct and indirect manner, as is easily seen in Eq. (10).

We know from these considerations that the resulting low energy values of λ and u are rather different in both models. We should note that these values affect the upper bound of the lightest neutral Higgs scalar mass. Although Eq. (7) shows λ is crucial to determine the tree level bound, u is essential to determine the magnitude of the one-loop effect, especially in the extra U(1) models. The radiative correction to Eq. (7) can be taken into account based on the one-loop effective potential. It is well-known that the one-loop contribution to the effective potential can be written as [19, 20]

$$V_1 = \frac{1}{64\pi^2} \text{Str } \mathcal{M}^4 \left(\ln \frac{\mathcal{M}^2}{\Lambda^2} - \frac{3}{2} \right), \quad (13)$$

where \mathcal{M}^2 is a matrix of the squared mass of the fields contributing to the one-loop correction and Λ is a renormalization point. In the usual estimation of the lightest neutral Higgs mass in the NMSSM the top and stop contributions to V_1 are mainly considered as the relevant fields because of their large Yukawa coupling. However, in the study of the extra U(1) models k is rather large and then we should also take account of the effect on \mathcal{M}^2 from the extra singlet colored chiral superfields g and \bar{g} which have a coupling with

S. A mass matrix of the stops is written as

$$\begin{pmatrix} \tilde{m}_Q^2 + h_t^2 v_2^2 & h_t v_2 (-A_t + \lambda u \cot \beta) \\ h_t v_2 (-A_t + \lambda u \cot \beta) & \tilde{m}_{\bar{T}}^2 + h_t^2 v_2^2 \end{pmatrix}, \quad (14)$$

and the one of the s-gquarks is expressed as

$$\begin{pmatrix} \tilde{m}_g^2 + k^2 u^2 & -A_k k u + \lambda k v_1 v_2 \\ -A_k k u + \lambda k v_1 v_2 & \tilde{m}_{\bar{g}}^2 + k^2 u^2 \end{pmatrix}, \quad (15)$$

where $\tilde{m}_{Q,\bar{T}}^2$, $\tilde{m}_{g,\bar{g}}^2$ and A_t, A_k are soft supersymmetry breaking parameters. Here a D-term contribution is neglected as it has been done in many previous investigations of the MSSM [2]. Mass eigenvalues of these mass matrices are respectively expressed as,

$$\begin{aligned} \tilde{m}_{t_i}^2 &= \frac{1}{2}(\tilde{m}_Q^2 + \tilde{m}_{\bar{T}}^2) + h_t^2 v_2^2 \pm \sqrt{\frac{1}{4}(\tilde{m}_Q^2 - \tilde{m}_{\bar{T}}^2) + h_t^2 v_2^2 (-A_t + \lambda u \cot \beta)^2}, \\ \tilde{m}_{g_i}^2 &= \frac{1}{2}(\tilde{m}_g^2 + \tilde{m}_{\bar{g}}^2) + k^2 u^2 \pm \sqrt{\frac{1}{4}(\tilde{m}_g^2 - \tilde{m}_{\bar{g}}^2) + (-A_k k u + \lambda k v_1 v_2)^2}. \end{aligned} \quad (16)$$

If we estimate the upper bound of the lightest Higgs mass in the same procedure as the one used to obtain Eq. (7) by minimizing the one-loop effective potential $V_{\text{eff}} = V_0 + V_1$, the following one-loop correction should be added to the right-hand side of Eq. (7):

$$\Delta m_{h^0}^2 = \frac{1}{2} \left(\frac{\partial^2 V_1}{\partial v_1^2} - \frac{1}{v_1} \frac{\partial V_1}{\partial v_1} \right) \cos^2 \beta + \frac{1}{2} \frac{\partial^2 V_1}{\partial v_1 \partial v_2} \sin 2\beta + \frac{1}{2} \left(\frac{\partial^2 V_1}{\partial v_2^2} - \frac{1}{v_2} \frac{\partial V_1}{\partial v_2} \right) \sin^2 \beta. \quad (17)$$

From these we find that u can crucially affect to the mass bound through the one-loop effect of g quark sector in the extra U(1) models. This additional effect cannot be escapable as far as the occurrence of the radiative symmetry breaking is required.

It may also be important to take account of the difference in both models coming from some phenomenological constraints, in particular, the ones related to λ and u . Although this kind of constraints depend on the values of soft supersymmetry breaking parameters, it may be useful to improve the upper bound estimation based on the triviality bound of λ . We should remind the fact that the chargino mass, the charged Higgs mass and squark masses are dependent on λ and u [11]. The chargino and the charged Higgs scalar have the same constituents as the MSSM. However, they have a different mass formulas from the MSSM [12]. In both models the chargino mass is expressed as

$$\begin{aligned} m_{\chi^\pm} &= \frac{1}{2} (\lambda^2 u^2 + 2m_W^2 + M_2^2) \\ &\pm \sqrt{\frac{1}{4} (2m_W^2 \cos 2\beta + \lambda^2 u^2 - M_2^2)^2 + 2m_W^2 (-\lambda u \sin \beta + M_2 \cos \beta)^2}, \end{aligned} \quad (18)$$

where m_W and M_2 represent the W boson and the gaugino λ_2^\pm masses. The charged Higgs scalar mass has the different mass formula between both models. In the extra U(1) models it is expressed as

$$m_{H^\pm}^2 = m_W^2 \left(1 - \frac{2\lambda^2}{g_2^2} \right) + \frac{2A_\lambda \lambda u}{\sin 2\beta}, \quad (19)$$

while in the NMSSM it is written as

$$m_{H^\pm}^2 = m_W^2 \left(1 - \frac{2\lambda^2}{g_2^2} \right) + \frac{2(A_\lambda \lambda u - \kappa \lambda u^2)}{\sin 2\beta}. \quad (20)$$

Recently the lower bounds of these masses become larger and we may use these to put some constraints on λ and u . Another important point to use Eq. (7) is that it must be smaller than other two diagonal mass matrix elements of the 3×3 neutral Higgs scalar mass matrix. Especially the diagonal mass for the singlet Higgs scalar S can give a substantial constraint on u . Its tree level formula is

$$m_{H_3^0}^2 = \frac{1}{2} g_E^2 Q_S^2 u^2 + \frac{A_\lambda \lambda v_1 v_2}{u} \quad (21)$$

in the extra U(1) models, while it is expressed as

$$m_{H_3^0}^2 = 4\kappa^2 u^2 + \frac{A_\lambda \lambda v_1 v_2}{u} - A_\kappa \kappa u \quad (22)$$

in the NMSSM. This constraint may be substantial in the NMSSM where there is no other clear constraint on the small u .

4. Numerical analysis and its results

In this section we numerically estimate the bound of $m_{h^0}^2 (\equiv m_{h^0}^{(0)2} + \Delta m_{h^0}^2)$ by solving the RGEs and taking account of the phenomenological constraints presented above. In order to improve the one-loop effective potential [21] We use two-loop RGEs for dimensionless coupling constants and one-loop ones for dimensional SUSY breaking parameters, for simplicity. In this estimation we adopt the following procedure. As the initial conditions for the SUSY breaking parameters we take

$$\tilde{m}_{\phi_i}^2 = (\gamma_i \tilde{m})^2, \quad M_a = M, \quad A_t = A_k = A_\kappa = A_\lambda = A, \quad (23)$$

where \tilde{m}^2 is the universal soft scalar mass and we introduce the nonuniversality represented by γ_i only among soft scalar masses of H_1, H_2 and S . We comment on this point

later. These initial conditions are assumed to be applied at the scale where the coupling unification of $SU(2)_L$ and $U(1)_Y$ occurs. We donot require the regolous coupling unification of $SU(3)_C$ but only impose the realization of the low energy experimental value following Ref. [16]. For the extra $U(1)$ coupling g_E we use the same initial value as the one of $U(1)_Y$ at the unification scale M_X . The initial values of these parameters are surveyed through the following region,

$$\begin{aligned} 0 \leq h_t \leq 1.2 \quad (0.1), \quad 0 \leq k, |\kappa| \leq 2.0 \quad (0.2), \quad 0 \leq \lambda \leq 3.0 \quad (0.2), \\ 0 \leq M/M_S \leq 0.8 \quad (0.2), \quad 0 \leq \tilde{m}/M_S, |A|/M_S \leq 3.0 \quad (0.3), \end{aligned} \quad (24)$$

where in the parentheses we give the interval which we use in the survey of these parameter regions. Since the sign of κ and A affect the scalar potential, we need to investigate both sign of them. We also assume that the RGEs of the model are changed from the ones of the supersymmetric extra $U(1)$ models to the nonsupersymmetric ones at a supersymmetry breaking scale M_S for which we take $M_S = 1$ TeV as a typical numerical value [14, 16]⁷.

As a criterion for the choice of the correct vacuum, we impose that the radiative symmetry breaking occurs correctly. We check whether the potential minimum satisfying the conditions such as Eq. (6) improved by the one-loop effective potential can satisfy the phenomenologically required conditions such as $v = 174$ GeV and $m_t = 174$ GeV starting from the above mentioned initial conditions. It is not so easy to find this solution under the completely universal soft breaking parameters so that in our RGEs analysis we allow the nonuniversality in the region $0.8 \leq \gamma_i \leq 1.2$ among soft supersymmetry breaking masses of Higgs scalars. The nonuniversality of soft scalar masses are generally expected in the superstring models [22]. This treatment seems to be good enough for our purpose such as to estimate the upper mass bound of Higgs scalar. We also additionally impose the following phenomenological conditions.

- (i) $m_{h^0}^2$ should be smaller than other diagonal components of the Higgs mass matrix (see also footnote 3 and the discussion related to Eqs. (13) and (14)).
- (ii) the experimental mass bounds on the charged Higgs bosons, charginos, stops, gluinos

⁷In principle we should solve the RGEs of soft supersymmetry breaking parameters under the initial values given in Eq. (15) in order to estimate this scale M_S . However, we donot take such a way here, for simplicity. It is beyond the present scope to study the dependence of our results on the supersymmetry breaking scale M_S .

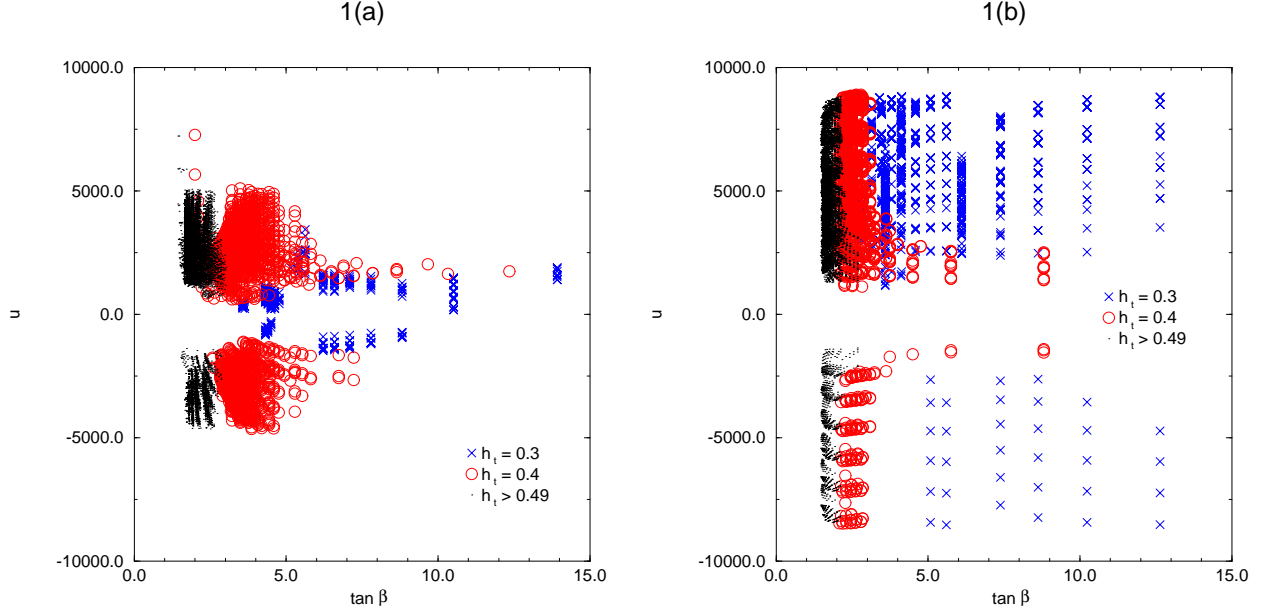


Fig.1 Scatter plots of the radiative symmetry breaking solutions in the $(\tan\beta, u)$ plane for the NMSSM 1(a) and the ξ_- model 1(b). Solutions for the different $h_t(M_X)$ are classified.

and Z' should be satisfied. Here we require the following values:

$$\begin{aligned}
 m_{H^\pm} &\geq 67 \text{ GeV}, & m_{\chi^\pm} &\geq 72 \text{ GeV}, & \tilde{m}_{t_{1,2}} &> 67 \text{ GeV}, \\
 M_3 &\geq 173 \text{ GeV}, & m_{Z'} &\geq 500 \text{ GeV}.
 \end{aligned}
 \tag{25}$$

(iii) the vacuum should be a color conserving one [23].

We adopt only the parameters set satisfying these criterions as the candidates of the correct vacua and calculate the Higgs mass bound $m_{h^0}^2$ for them.

At first in order to see the difference in the allowed vacuum between the NMSSM and the extra U(1) models we plot the radiative symmetry breaking solutions for the present parameter settings in the $(\tan\beta, u)$ plane in Fig. 1. Solutions are classified by the initial value of h_t at M_X into three classes which show rather different qualitative features. As an example of the extra U(1) models we take the ξ_- model here but the η model has been checked to show the similar feature to the ξ_- model. We take the case (A) as the extra matter contents. Throught the present calculation an effect of the translation of the running mass to the pole mass [24] is taken into account to determine $\tan\beta$. We take $\tan\beta \leq 15$ and neglect the large $\tan\beta$ solutions since the bottom Yukawa coupling is assumed to be small in the RGEs so that in the present analysis the large $\tan\beta$ solutions

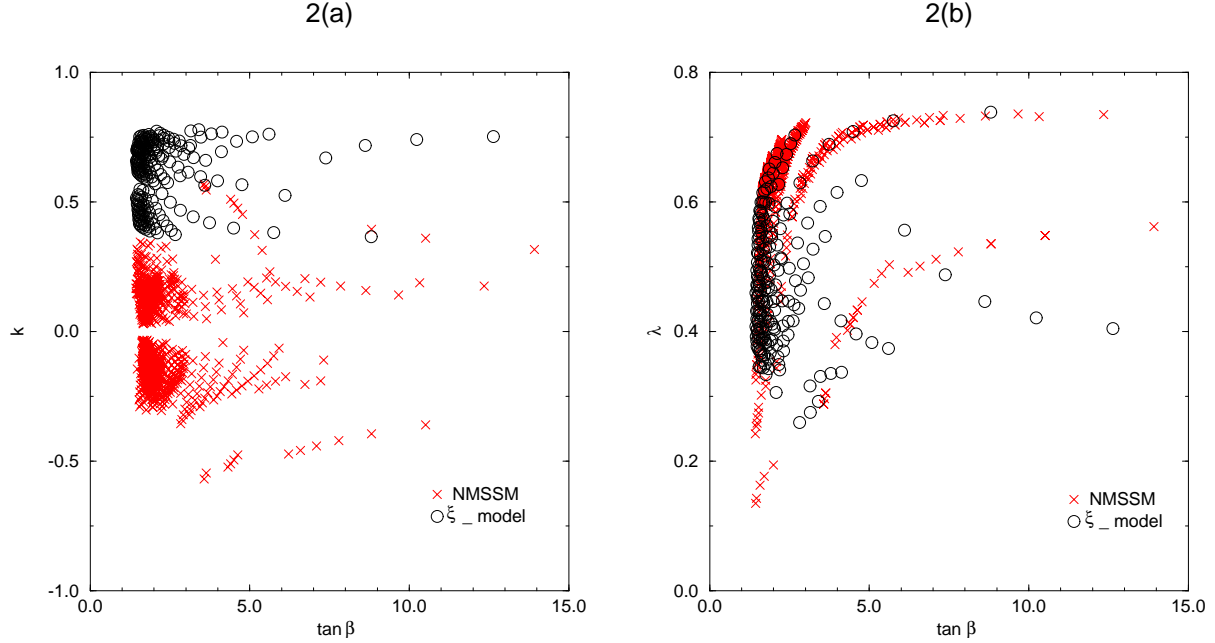


Fig.2 Scatter plots of the radiative symmetry breaking solutions for the NMSSM and the ξ_- model in the $(\tan\beta, k$ or $\kappa)$ plane 2(a) and the $(\tan\beta, \lambda)$ plane 2(b). The values of k , κ and λ are the ones at m_t .

cannot be recognized as the appropriate ones. Figure 1 shows that the ξ_- model can have solutions in the larger u region of the $(\tan\beta, u)$ plane compared with the NMSSM. As mentioned in the previous section, this is a result that k can be larger than κ at the m_t scale due to the SU(3) effect. This is shown in Fig.2, where the values of $k(m_t)$, $\kappa(m_t)$ and $\lambda(m_t)$ corresponding to each solution are plotted for $\tan\beta$. The soft scalar mass m_S^2 of the singlet Higgs scalar S becomes much more negative in the extra U(1) models than in the NMSSM. In the sufficiently large u region the potential minimum condition for u reduces to

$$u^2 = -\frac{4m_S^2}{g_E^2 Q_S^2} \quad \text{for extra U(1)}, \quad u^2 = -\frac{m_S^2}{2\kappa^2} \quad \text{for NMSSM}. \quad (26)$$

In the NMSSM u depends not only on m_S^2 but also on κ and as a result u can take a rather large value. In the ξ_- model the smaller u region such as $u \lesssim 1$ TeV is cut due to the experimental extra Z mass bound. Also in the NMSSM very small u seems to be forbidden. This seems to be a result of the phenomenological conditions (i) and (ii).

The big qualitative difference of the vacuum in both models is that there can be large u solutions for $\tan\beta \gtrsim 5$ corresponding to $h_t(M_X) = 0.3$ in the extra U(1) model. One reason of this is that the smaller $\lambda(m_t)$ is realized in the extra U(1) models than in the

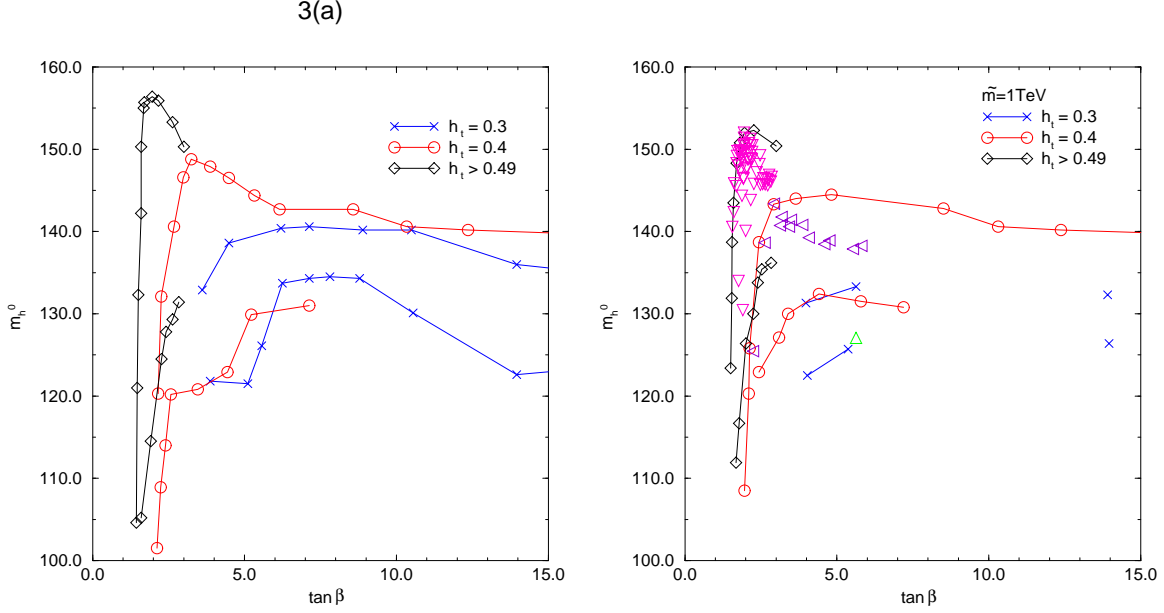


Fig.3 Boundary values of m_{h^0} of the lightest neutral Higgs mass as a function of $\tan \beta$ in the NMSSM. Full data are used to draw 3(a). In 3(b) we impose $\tilde{m}(M_X) = 1 \text{ TeV}$. All solutions satisfying $2.4 \text{ TeV} \leq u \leq 2.6 \text{ TeV}$ are also plotted in 3(b).

NMSSM. This is clearly shown in Fig. 2(b). The discussion on this aspect has already given based on the RGE in the previous section. On this point we should also note that in the $\tan \beta \gtrsim 5$ region the small $\lambda(m_t)$ is allowed. Thus $\mu = \lambda u$ can be in the suitable range even if u is large. However, the boundary value of u seems not to have so strong dependence on $\lambda(m_t)$ in both models and the value of λu does not seem to be strictly restricted by the radiative symmetry breaking at least within the parameter region searched in this paper.

In Figs. 3~5 we give the results of our numerical estimations of m_h^0 for each model. In these figures we plot the boundary values of m_h^0 for the parameters obtained as the solutions of our radiative symmetry breaking study. In each figure (a) the upper and lower boundaries of m_h^0 are drawn by using the all solutions obtained under the initial values shown in (24). In order to show the $h_t(M_X)$ dependence of m_h^0 we classify the solutions into three classes and draw them separately. In figures (b) we plot the upper and lower boundaries of m_h^0 for the remaining solutions after imposing the additional condition $\tilde{m} = 1 \text{ TeV}$. We also add the scatter plots of the all solutions corresponding to $2.4 \text{ TeV} < u < 2.6 \text{ TeV}$ in the same figures. They are represented by three kinds of

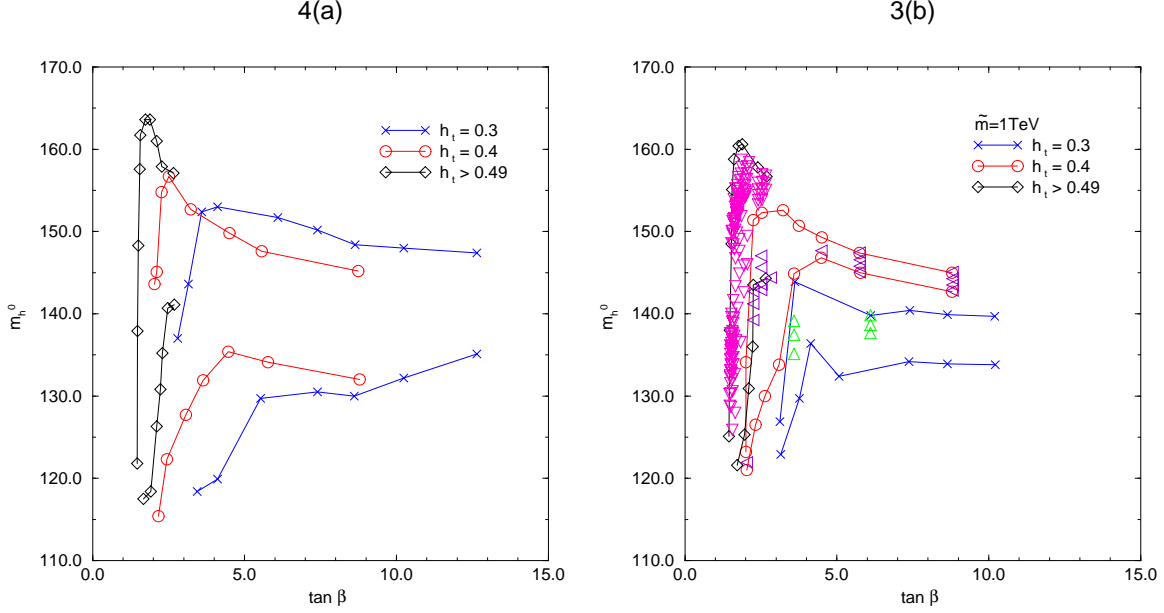


Fig.4 Boundary values of m_{h^0} of the lightest neutral Higgs mass as a function of $\tan \beta$ in the ξ_- model. Full data are used to draw 4(a). In 4(b) we impose $\tilde{m}(M_X) = 1$ TeV. All solutions satisfying $2.4 \text{ TeV} \leq u \leq 2.6 \text{ TeV}$ are also plotted in 4(b).

triangles corresponding to each $h_t(M_X)$. As a common feature in all models, we find that the larger $h_t(M_X)$ realizes the smaller $\tan \beta$ and then brings the larger contribution of the second term of Eq. (7). Thus the largest $\lambda(m_t)$ in the small $\tan \beta$ in Fig. 2(b) gives the largest m_h^0 . Although $\lambda(m_t)$ in the extra U(1) models can be smaller than the one of the NMSSM as shown in Fig. 2(b), the boundary values of m_h^0 is larger in the extra U(1) models than in the NMSSM by a few to ten GeV. This is mainly due to the extra contribution to Eq. (17) coming from the singlet colored fields (g_i, \bar{g}_i) . Since the existence of this contribution is the basic feature of the present extra U(1) models, the boundary value of m_h^0 is generally expected to be larger than the one of the NMSSM inspite of the running feature of the Yukawa coupling λ . This one-loop effect is large enough to cancel the difference of $\lambda(m_t)$ in the second term of Eq. (7). In our studying parameters space the largest value of m_h^0 is

$$m_h^0 \lesssim 156 \text{ GeV (NMSSM)}, \quad m_h^0 \lesssim 164 \text{ GeV (NMSSM)}, \quad m_h^0 \lesssim 158 \text{ GeV (NMSSM)}. \quad (27)$$

By comparing (a) and (b) in Figs. 3~5 we can get the tendency how the solutions are restricted when we reduce the parameter space. The change of \tilde{m} and u mainly affect the one loop contribution through the mass matrices (14) and (15).

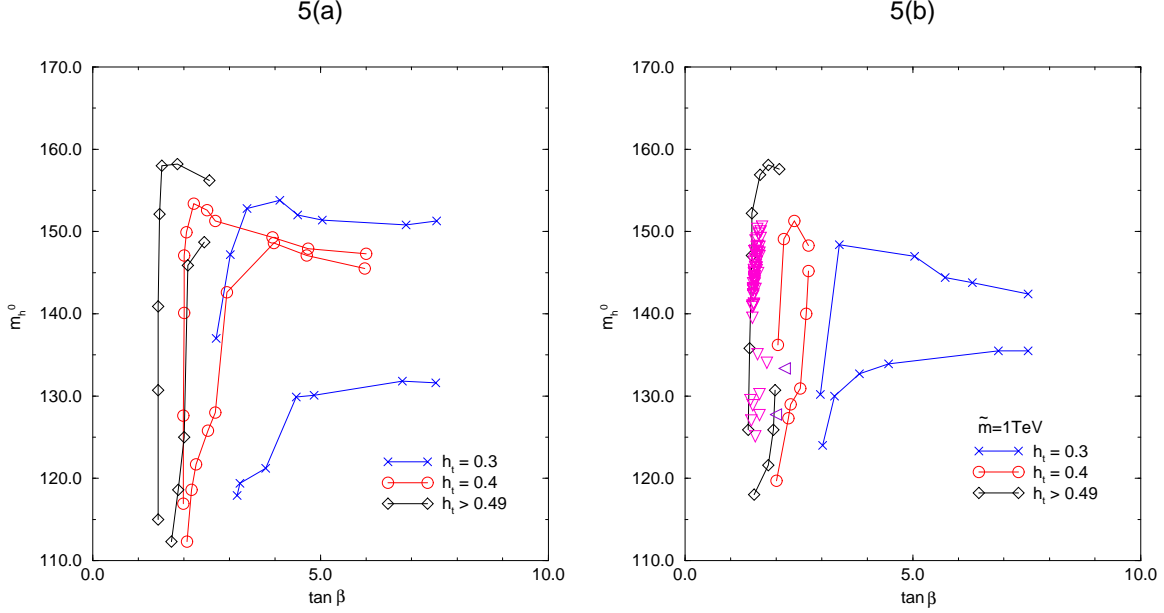


Fig.5 Boundary values of m_{h^0} of the lightest neutral Higgs mass as a function of $\tan\beta$ in the η model. Full data are used to draw 5(a). In 5(b) we impose $\tilde{m}(M_X) = 1$ TeV. All solutions satisfying $2.4 \text{ TeV} \leq u \leq 2.6 \text{ TeV}$ are also plotted in 5(b).

In Fig. 6 we plot the boundary value of m_h^0 for u in the ξ_- model. This shows the tendency that the larger u gives the larger value of m_h^0 . This is expected from the one-loop contribution of the extra singlet colored fields (g_i, \bar{g}_i). From this figure we can read off the relation between $m_{Z'}$ and m_h^0 by using $m_{Z'}^2 \sim g_E^2 Q_S^2 u^2 / 2$. The lower bound of $m_{Z'}$ in Fig. 6 is about 600 GeV where we used $g_E(m_t) = 0.36$. The conditions (i) and (ii) also determine the lower bound of u in the extra U(1) models.

Finally we give a few comments on some points related to the extra matters. We also studied the case (B) of the extra matter contents for the same parameter settings as the above study. In that case, as a common feature we can find, it becomes rather difficult to satisfy both of the radiative symmetry breaking conditions and the phenomenological conditions (i) to (iii) compared with the case (A). The number of solutions in the case (B) is drastically less than in the case (A). Since the value of $g_3(M_X)$ increases, $h_t(m_t)$ and $k(m_t)$ becomes larger. In fact, the initial value of h_t in the wide region such as $0.2 \leq h_t(m_t) \leq 0.9$ results in only the small $\tan\beta$ (larger $h_t(m_t)$) solution such as $\tan\beta \lesssim 1.8$. This also makes $\lambda(m_t)$ smaller. The larger $\tan\beta$ solutions disappear and the value of $|u|$ is shifted upward. However, the upper boundary value of m_h^0 behaves in the different way

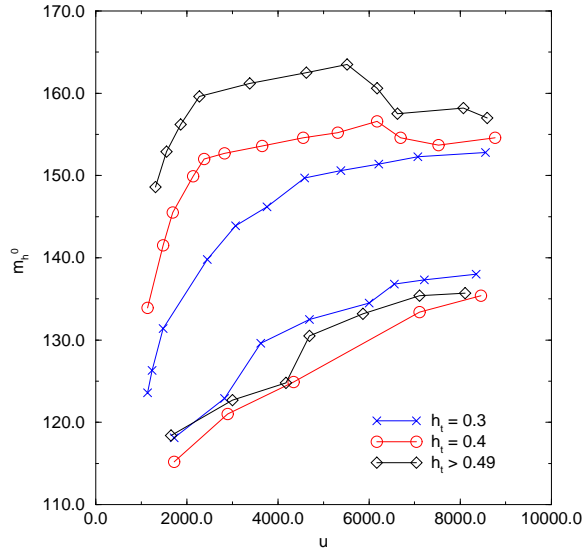


Fig.6 Boundary values of m_{h^0} of the lightest neutral Higgs mass as a function of u in the ξ_- model.

between the NMSSM and the extra U(1) models. Although in both models m_{h^0} becomes smaller in the region of $\tan\beta > \sim 2$, the behavior is different at $\tan\beta < \sim 2$. In the NMSSM it is a little bit larger than the one of case (A). On the other hand, it becomes smaller than the one of case (A) by a several GeV in the extra U(1) models. Here we should remind the fact that even if $\lambda(m_t)$ is smaller m_{h^0} can be larger in the case that corresponding $\tan\beta$ is smaller. The difference in the RGE of λ in both model is also important in this behavior. To have more confident quantitative results in this case we need to search the parameter space in the finer way. We also changed the number of (g_i, \bar{g}_i) which couples to S in the superpotential (1) in the case (A). If we decrease this number from three to one, the boundary values of the allowed m_h^0 become larger. This reason is considered as follows. Although this decrease reduces the number of fields contributing to the one-loop effective potential, this also decreases the N_g value in Eq. (10). As a result the larger k and λ are realized at the low energy region. The larger k also brings the larger u . The contribution to the one-loop effect per a field can be larger. Thus the decrease of the number of (g_i, \bar{g}_i) which couples to S causes the increase of m_h^0 at not only the tree level but also the one-loop level.

5. Summary

There are two well-known low energy candidates to solve the μ problem in the MSSM. These are the NMSSM and the extra U(1) models. We have estimated the upper bound of the lightest neutral Higgs mass in both models. Apart from a Higgs coupling $\lambda SH_1 H_2$, there is a typical coupling κS^3 in the NMSSM and $kSg\bar{g}$ in the extra U(1) models. In the NMSSM κ plays a crucial role in the evolution of λ which dominantly determines the tree level mass bound of the lightest neutral Higgs scalar and in the radiative symmetry breaking. In the extra U(1) models the introduction of the extra colored fields g, \bar{g} and its coupling with the singlet Higgs S are crucial to cause the radiative symmetry breaking at the weak scale successfully. This coupling can also affect the running of the coupling constant λ . We focussed our attention on these points and estimated the the upper bound of the lightest neutral Higgs mass in both models. In this estimation we additionally imposed some phenomenological constraints related to λ and the VEV of S coming from, for example, the mass bounds of the charginos, the charged Higgs scalars and the Z' boson. We solved the minimum conditions of the one-loop effective potential improved by the RGEs for the couplings and soft supersymmetry breaking parameters whose initial conditions are taken in the suitable region. We estimated the upper bound of the lightest neutral Higgs scalar for the parameters which bring the phenomenologically correct potential minimum. Its tree level contribution due to λ can be smaller in the extra U(1) models than in the NMSSM. However, there is the extra one-loop contribution originated from the Yukawa coupling $kSg\bar{g}$ and this makes its upper bound larger in the extra U(1) than in the NMSSM by a few to ten GeV. It is interesting enough that the upper bound of the lightest neutral Higgs scalar in the extra U(1) models is not so different from the one of the NMSSM. The extra U(1) models may be an equal candidate to the NMSSM for the experimental Higgs search.

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